Mixing with time dependent natural convection

L.M. de la Cruz a,*, E. Ramos b

a Visualization Department, DGSCA, National University of Mexico
04510, México D.F., Mexico

b Center for Energy Research, National University of Mexico
Ap. P. 34, 62580, Temixco Mor., Mexico.

Abstract

Chaotic mixing inside a two dimensional cavity can be achieved with time dependent natural convection if the motion of a fluid is generated by imposing alternating hot and cold wall temperatures. With this set up no moving walls are required to mix the fluid inside the container. In this comment we illustrate this idea by numerically solving the governing equations of natural convection in a two dimensional square cavity with sections of its upper and lower horizontal walls cooled and heated in a periodic manner. These conditions generate a vortex of time dependent intensity that moves its center in a closed loop around the geometrical center of the container. The mixing properties of the flow are illustrated by Lagrangian tracking of a collection of points originally located in a line.

Key words: Mixing, natural convection

1 Introduction

There has been a recent surge of studies of fundamental properties of the mixing due to its applications in manufacturing, food, pharmacology and other industries. The literature reporting progress in the field is large and the interested reader is referred to monographs and review articles for a comprehensive survey. See for instance [1] and [2]. Here we shall comment only the papers that are directly relevant for the present study. A pioneer investigation on the...
subject is the blinking vortices flow proposed by Aref. This mixing flow is the two dimensional motion of an incompressible inviscid fluid generated by two corotating point vortices fixed in space that are switched alternatively on and off. This flow can be described analytically and mixing can be illustrated by Lagrangian tracking of markers whose position of markers at all times, can be determined numerically with a high degree of accuracy. By iterating the mapping of subsequent positions of markers for different values of the external parameters, the regimes of regular and chaotic behavior can be identified. In this example, the structure of periodic points and their bifurcations are well established [3].

Another mixing study that is particularly relevant for our analysis is the translating–rotating mixer introduced by Finn and Cox [4]. This mixer consists of a circular cylindrical vat inside which the fluid is stirred by a rod with circular cross section. The rod can be moved across the fluid and also can be rotated around its own axis. The dynamics of this system is similar to that of a vortex that moves describing a prescribed orbit inside the container. Various mixing protocols were analyzed and the mixing efficiencies in terms of energy input have been described.

Although several studies have addressed natural convection with time dependent wall temperatures or heat inputs, this phenomenon is far from being thoroughly understood. The mixing properties of time dependent natural convection have not been explored yet. Lelong and Bejan [5] present a numerical investigation of natural convection in a square enclosure with one isothermal vertical wall and the opposite wall heated with a pulsating heat flux. They found a resonance frequency at which a maximum Nusselt number was attained. For Pr = 7 and Ra = 10^7, the resonant frequency is approximately 1.5 times than the natural frequency of the convective cell. Performing intensive numerical calculations to study natural convection of a square cavity with a constant-temperature cold sidewall and an opposite hot sidewall with sinusoidally-varying temperatures, Kwak et al. [6], were able to determine the resonant frequency of the heated vertical wall where a maximum of heat transfer and convective velocity was attained. Their calculations agree with the suggestion given by Paolucci and Chenoweth [7] that indicates that the resonant frequency is \( \frac{1}{\sqrt{2}} f \) where \( f \) is the Brunt-Vaisala frequency. In the present article, we study a mixing flow produced by a time dependent wall temperature in presence of a body force. These conditions generate a vortex of variable strength whose center moves around the container. This mixing protocol can be interpreted as a combination of a translating rotating mixer with blinking vortices. These conditions generate chaotic mixing flows where no moving walls are required.
2 Theoretical model and analysis

Consider natural convection in a two dimensional square, with the temperature of the left half of the upper wall cyclically reduced and the right half of the bottom wall cyclically increased. Temperature changes are out-phased. The two vertical walls are adiabatic. The geometry of the cavity and the coordinate system are shown in figure 1. The Boussinesq approximation is assumed to be valid and the state equation considered is \( \rho = \rho_0 (1 - \beta (T - T_o)) \), where \( \beta \) is the volumetric expansion coefficient and the subindex \( o \) denotes a reference state.

The governing equations in terms of non-dimensional variables are:

\[
\nabla \cdot \mathbf{u} = 0, \tag{1}
\]
\[
\frac{\partial u}{\partial t} + \nabla (u \mathbf{u}) = - \frac{\partial p}{\partial x} + \text{Pr} \nabla^2 u, \tag{2}
\]
\[
\frac{\partial v}{\partial t} + \nabla (v \mathbf{u}) = - \frac{\partial p}{\partial y} + \text{Pr} \nabla^2 v + Ra \text{Pr} T, \tag{3}
\]
\[
\frac{\partial T}{\partial t} + \nabla (T \mathbf{u}) = \text{Pr} \nabla^2 T. \tag{4}
\]

The spatial coordinates and time are scaled with the length of the enclosure side, \( L \) and \( L^2/\alpha \) respectively. \( \alpha \) is the thermal diffusivity. \( \mathbf{u} = (u, v) \) is the velocity vector scaled with \( \alpha/L \), \( p \) is the pressure difference between the total pressure and the equilibrium hydrostatic in absence of a temperature gradient and scaled with \( \rho \alpha^2/L^2 \); \( \rho \) is the fluid density. The temperature \( T \) is nondimensionalized by \( (T-T_m)/\Delta T \), \( \Delta T = T_H - T_C \) and \( T_m = (T_H + T_C)/2 \). \( T_H \)
and $T_C$ are the maximum and minimum wall temperatures and $T_H = -T_C$. The Prandtl ($Pr$) and Rayleigh ($Ra$) numbers are defined by:

$$Pr = \frac{\nu}{\alpha} \quad \text{and} \quad Ra = \frac{g\beta\Delta TL^3}{\nu\alpha}.$$  

Here, the acceleration of gravity is $g$ and $\nu$ is the kinematic viscosity. The origin of the coordinate system is taken at the geometrical center of the cavity and boundary conditions are:

$$u = 0, \quad \frac{\partial T}{\partial x} = 0 \quad \text{for} \quad x = -0.5, 0.5 \quad \text{and} \quad -0.5 \leq y \leq 0.5, \quad (5)$$

$$u = 0, \quad T = 0 \quad \text{for} \quad -0.5 \leq x \leq 0 \quad \text{and} \quad y = -0.5, \quad (6)$$

$$u = 0, \quad T = f_1(t) \quad \text{for} \quad 0 \leq x \leq 0.5 \quad \text{and} \quad y = -0.5, \quad (7)$$

$$u = 0, \quad T = f_2(t) \quad \text{for} \quad -0.5 \leq x \leq 0 \quad \text{and} \quad y = 0.5, \quad (8)$$

$$u = 0, \quad T = 0 \quad \text{for} \quad 0 \leq x \leq 0.5 \quad \text{and} \quad y = 0.5. \quad (9)$$

The first cycle of the periodic functions $f_1$ and $f_2$ is defined by the following expressions:

$$f_1(t) = \begin{cases} 
0.5 \sin^2(4t) & \text{for} \quad 0 \leq t < \pi/8 \\
1 & \text{for} \quad \pi/8 \leq t < 7\pi/8 \\
0.5 \sin^2(4t - 3\pi) & \text{for} \quad 7\pi/8 \leq t < \pi \\
0 & \text{for} \quad \pi \leq t < 2\pi 
\end{cases}$$

and

$$f_2(t) = \begin{cases} 
0 & \text{for} \quad 0 \leq t < \pi \\
-0.5 \sin^2(4t - 3\pi) & \text{for} \quad \pi \leq t < 9\pi/8 \\
1 & \text{for} \quad 9\pi/8 \leq t < 15\pi/8 \\
-0.5 \sin^2(4t - 6\pi) & \text{for} \quad 15\pi/8 \leq t < 2\pi 
\end{cases}$$

Note that the relative phase of the wall temperature change is $\pi$.

The numerical solution of the conservation equations has been found using an Object-Oriented library based on the control volume method [8,9]. The library is a set of classes written in the C++ language, implemented using sophisticated techniques based on templates in order to obtain good performance.
Also, it contains classes for running in parallel architectures. The QUICK scheme was used for the convective terms and the SIMPLEX algorithm was used to solve the coupled equations. A grid independence analysis was carried out to obtain solutions to the balance equations using uniform grids. A staggered grid was used for all calculations and the time integration was accomplished with a backward Euler scheme and the uniform discretization mesh contains $256^2$ control volumes.

### 3 Results and discussion

The oscillatory wall temperature imposed on the left half of the top wall and the right half of the bottom wall yields the formation of alternating ascending and descending thermal plumes in regions close to the vertical walls. In turn, these structures induce a global motion that displays an initial transient followed by a periodic motion. All results presented below, refer to the periodic motion that occurs once the initial transient has died out. In some events that take place in this periodic flow, it is more convenient to refer to the phase in the cycle ($\phi$) rather to the absolute time. In all figures, $Pr=5$ and $Ra=10^5$. The evolution of the temperature field inside the cavity is illustrated in figure 2 where the temperature field is plotted as a function of position for four phase angles. The upward and downward thermal plumes are shown at $\phi = \pi/2$ and $\phi = 3\pi/2$ respectively. The phase $\phi = \pi/2$, corresponds to half the interval of high temperature at the lower wall and the ascending plume is in the process of formation. At $\phi = \pi$, the temperature of the wall levels and the first half of the cycle ends. The phase with low temperature at the left half of the upper wall, which comprises the second half of the cycle, displays analogous features and is not shown.

The major characteristics of the convective motion are shown in figures 3, where the velocity field is plotted as a function of position at equally spaced times in one cycle. At the beginning of a cycle, the hot thermal plume forms and subsequently, collides with the top of the cavity. Then, it turns left toward the interior of the cavity and slowly dissipates. The ascending plume of hot fluid, and its interaction with the top wall, generates a vortical structure in the core of the cavity. The vortex is not centered and presents a narrow region with large ascending velocity near the right wall and a wider region with slower velocity near the left wall. In the second part of the cycle, a flow with corresponding features is generated by the descending plume. See figure 3. The velocity and temperature fields display the following symmetry:
Due to the periodic nature of the formation of the plumes, and their asymmetric position, the center of the vortex defined by the point with zero velocity, describes a curve that roughly encircles the center of the cavity. The trajectory of the center of the vortex is also shown in figures 3 and its instantaneous position is denoted by the dots. The orbit of the vortex center is a curve akin to an epitrochoid with a base curve related to a rectangle. The vortex moves with short spells of large velocities followed by long spells of small velocities. In the loop eyes, the vortex moves with the lowest speeds.

In order to assess the intensity of the vortex, we calculated the total kinetic energy of the flow as a function of time $K(t)$. This variable, normalized with its average value is displayed in figure 4. The maxima of the kinetic energy occur at approximately half the high and low temperature intervals while the minima take place at the beginning of the constant temperature intervals. Detailed analysis of the evolution of the kinetic energy, shown in figure 5, reveals

\[
\begin{align*}
    u(x, y, t) &\rightarrow u(-x, -y, t + \pi), \\
v(x, y, t) &\rightarrow v(-x, -y, t + \pi), \\
T(x, y, t) &\rightarrow -T(-x, -y, t + \pi).
\end{align*}
\]
that at approximately half the high temperature period, a high kinetic energy region forms near the right vertical wall. This event triggers the formation of two other clearly identifiable regions of smaller but substantial kinetic energy.

Fig. 3. Left column, velocity fields; right column, kinetic energy for a), b) $\phi = 0$; c), d) $\phi = \pi/2$; e), f) $\phi = \pi$; g), h) $\phi = 3\pi/2$. In the right column, red and blue are high and low kinetic energy respectively. The line is the trajectory of the center of the vortex and the dot is the instantaneous position of the center of the vortex.
content. One of them occurs ahead and the other behind the region with the highest kinetic energy close to the top and bottom walls respectively. See figure 5 b). At approximately $5\pi/4$, a large fraction of the kinetic energy dissipates and a local minimum energy occurs. See figure 5 d). Fourier analysis indicates that the total kinetic energy is a periodic function with three characteristic frequencies. The Fourier component with the largest amplitude presents the smallest frequency and corresponds to twice the forcing frequency. This has a simple interpretation, since the two effects, high and low wall temperatures augment the kinetic energy of the system. The second and third Fourier components have half and one twentieth of the amplitude of the largest component respectively and occur due to the interaction of the vortex with the walls and with other parts of fluid not participating in the main structure. These components have two and three times respectively, the frequency of the dominant component.

![Graph](image)

Fig. 4. Kinetic energy of the flow normalized with its average value as a function of time. The black dots indicate the times at which the kinetic energy distribution is displayed in figure 5.

In summary, the overall effect of the alternating cold and hot plumes, produced with the protocol displayed in figure 1 in one full cycle, is the generation of a counterclockwise vortex with its center rotating around the geometrical center of the cavity in an orbit having more than two characteristic frequencies and maximum intensity occurring twice each cycle. This is the combination of blinking vortices with the translating–rotating mixer.

The mixing efficiency of the flow can be qualitatively assessed from the simultaneous Lagrangian tracking of a set of points. Figures 6 display the positions of $5 \times 10^4$ points originally located at a horizontal line at $y = 0.5$ are shown
Fig. 5. Kinetic energy as a function of position for times where extrema of total kinetic energy occur, as displayed in figure 4. a) \( \phi = 0.24\pi \), b) \( \phi = 0.56\pi \), c) \( \phi = 0.96\pi \), d) \( \phi = 1.24\pi \), e) \( \phi = 1.56\pi \), f) \( \phi = 1.96\pi \). The center of the vortex is the dip near the center of the kinetic energy distribution.

after 1/4, 1/2, 3/4, 1, 5 and 50 cycles. The mixing process consists on the alternating stretching and folding. The local stretching occurs mostly near the vertical walls where the largest stresses are generated by the convective plumes. In turn, the folding takes place near the horizontal walls where the fluid turns. With this technique, stretching events can be identified by the separation of points that were originally close to each other. After 5 cycles, the collection of points are spread across most of the area of the cavity but leaving “isles” without points. The isles are less noticeable after 50 cycles, indicating a better mixing, but it is not clear when (or whether) a uniform mixing will be achieved.
Fig. 6. The position of $5 \times 10^4$ points originally located at a horizontal line at $y = 0.5$ is shown after a) 1/4, b) 1/2, c) 3/4, d) 1, e) 5 and f) 50 cycles.

4 Conclusions

It has been demonstrated with an example that mixing in cavities can be achieved by changing the temperature of vertical sidewalls in an appropriate
manner. This method takes advantage of the physical properties of the fluid to generate the motion. This mixing method does not require moving external parts, thus preventing leaking and material loss in a very natural way. In the present paper the detailed analysis was made of only one flow with Pr=5 and Ra =10^5. In future work, the efficiency of mixing will be made as a function of Pr, Ra, distribution of wall temperature and geometry. Another aspect of the problem that requires a thorough analysis is the dynamic behavior after hundreds of cycles. This study is not simple, not only for the extremely long computing times required, but also because the error propagation may put very stringent demands on the accuracy of the calculations.

5 Acknowledgments

This work has been done with the financial support of CONACyT (Mexico) through project No. U41347-F. LM de la Cruz wishes to acknowledge the grants from the UNAM Computation Science and Engineering Postgraduate Program.

References


